1 Electric Current

In electrostatic we have consider electric charges at rest and the forces acting between them. We shall now consider the charges in motion this constitute electric current.

An electric current consist of moving charges. The electric current which is defined as rate of flow of charege or it is the time rate at which charge moves through specified surface of conductor. ~~or space.~~

If a charge q ~~is flows~~ moves in time t through specified surface of conductor then current I in the conductor is given by

$$\text{current} = \frac{\text{charge}}{\text{time}}$$

$$I = \cancel{\text{charge}}\ \frac{q}{t}$$

Depending upon the state of matter different types of conduction are

(I) Metallic conduction
(II) conduction in semiconductors and crystals
(III) Electrolytic conduction
(IV) gaseous conduction
(V) Plasma conduction

In the case of metallic conduction free electrons which is known as conduction electrons are

the charge carriers while in the case of semiconductor we have two types of charge carriers i.e electrons and holes. In the electrolytic conduction positive and negative ions are the charge carriers this is characterised by transfer of mass from one place to another and chemical change.
Similarity in the gaseous conduction positive ions and electrons take part under the high electric field.

A plasma is a new forth state of matter consist of electrons, positive ~~and~~ and negative ions, neutral atoms and molecules of very low density behaving as a gas but its properties are very different from those of gas because of its charge constituents which give high electric conductivity to it.

We are mainly concerned with current in metallic conduction i.e electron conduction. A current in the metallic conductor such as copper wire consist of flow of electrons from one part to other and is the time rate at which electrons pass through any cross-section of the wire

The unit of current is coulom/second which is called as ampere and is denoted by 'A'

$$\therefore 1 \text{ ampear} = \frac{1 \text{ coulomb}}{1 \text{ second}}$$

when one coulomb of charge passes through any cross-section in one second, a current of 1 ampere is said to pass through the cross section.

~~The direction of current I is taken~~

The direction of flow of current was taken from positive point to negative point of conductor is the conventional current direction

Steady and varying current

when the charges moves in material medium crossing some surface, current is generated and the value of current is total charge flowing through the surface per second.

If the direction of flow of charge and rate of flow remains constant with time then the current is known as steady current

A change in the direction of flow of charge or in the rate of flow of charge or in both with time make the current as a varying current. If the ~~vary~~ variation in current are the simple harmonic in nature then the current is called as varying currents

Smaller unit of current in practical use are milliamperes (mA) and microamperes (μA)

$$1\,A = 1000\,mA = 10^3\,mA$$

$$1\,A = 1{,}000{,}000\,\mu A$$

$$= 10^{+6}\,\mu A$$

The alphabet I is generally use to represent current. The direction of current is shown by the arrow.

Current density

The current I is characteristics of particular conductor, as it is the total charge passing through the conductor of per unit time across any cross-section. therefore current can be determined by the total charge that flow through the conductor. the total charge passing through the every element of cross-section is same or not. For this we introduced microscopic concept of current density.

* If the charge passing through the every element of cross section of conductor is not same, we introduce microscopic parameter called current density J at every point of conductor. so the current density J is characteristics of a point inside a conductor.

consider a conductor of a cross section 'A' carrying steady current I. Assuming current is uniformly distributed on that cross section then magnitude J of the current density for all points on that cross section is

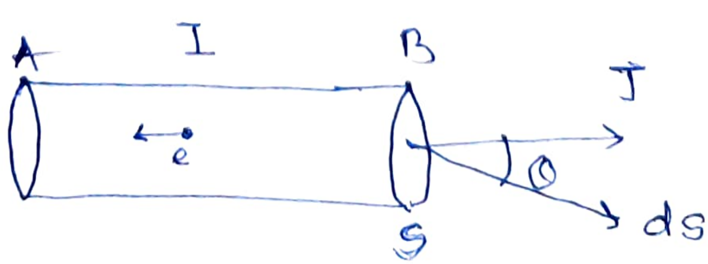
$$J = \frac{I}{A} \quad \text{--- (1)}$$

The vector J at any point is oriented in the direction that the positive charge would move at the point. An electron at that point would move in the direction of $-J$.

If the J is varies from point to point then total current is given by

$$I = \int_S J \cdot ds \quad \text{--- (2)}$$

Here the integration is performed over the area S. This expression shows that current density may be regarded as the flux of J.



Equation of continuity

The relation between current and current density is given by eqn (2)

$$I = \int_S J \cdot ds \quad \text{------------} (i)$$

consider a closed surface enclosing volume V.
if ρ is the charge density for very small volume dv. Then total charge in volume V is $\int_V \rho \, dv$
where ρ is the volume charge density

$\therefore$ Rate of decrease of charge in volume is

$$-\frac{\partial}{\partial t} \int_V \rho \, dv$$

According to the law of conservation of charge, rate of flow of charge through the enclosed volume is equal to rate of decrease of charge in it

$$\int_S J \cdot ds = -\frac{\partial}{\partial t} \int_V \rho \, dv$$

$$= -\int_V \frac{\partial \rho}{dt} \, dv$$

Since time and space coordinate are indepen-dent to each other

by using Gauss divergence theorem

to convert surface integral to volume

intergral

$$\int_V \mathrm{div}\, J \, dV = - \int_V \frac{d\rho}{dt} \, dV$$

$$\text{or } \mathrm{div}\, J = - \frac{d\rho}{dt}$$

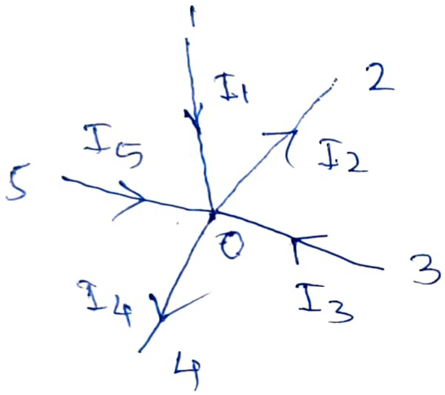
$$\text{or } \mathrm{div}\, J + \frac{d\rho}{dt} = 0$$

Above equation is the required equation of continuity

Kirchhoff's Laws

For steady currents flowing through ~~the~~ a network of conductor the following two laws known as Kirchhoff's law are applicable.

① First law : The algebraic sum of currents meeting at any junction in a circuit is zero.



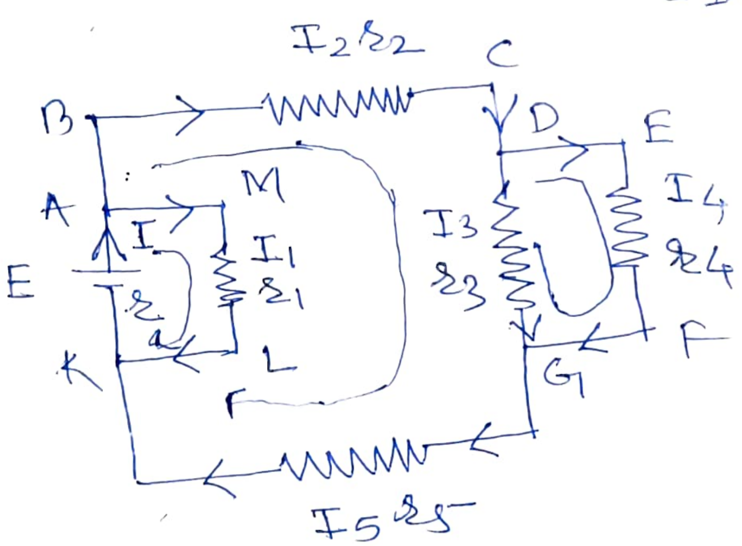
Let 1, 2 etc. be a conductors meeting at point O of an electrical circuit and I_1 I_2 etc be the current passing through them. Taking the current flowing towards the point as positive and those flowing away from the point is negative; the algebraic sum of currents which is $I_1 - I_2 + I_3 - I_4 + I_5 \cancel{=0}$ is equal to zero according to the first law. In general $\sum I = 0$

taking the current as the rate of flow of charge, the total flow of charge towards a point must be the same as the total out flow of charge in the same time as there is no accumulation of charge at any point in the ckt.

IInd law

In any closed path of an electrical ckt, the algebraic sum of the product of the currents and resistances of various branches of the path is equal to total emf of the path ckt.

$-Ir - I_2 r_2 - I_3 r_3 - I_5 r_5 - E = 0$



consider the electrical circuit given above the value of currents and resistances are indicated in the diagram.

Applying second law.

~~Appl.~~

(I) to the path $\underline{ABCDGKA}$

$\cancel{I r + I_2 r_2 + I_3 r_3 + I_5 r_5 = E}$

$-I_2 r_2 - I_3 r_3 - I_5 r_5 + E - I r = 0$

~~or $\sum I r = \sum E$~~

$E = I r + I_2 r_2 + I_3 r_3 + I_5 r_5$

Note for any path the product is taken as positive for the current in one direction and negative ~~f~~ in the opposite direction.
Further, if the direction of the current due to the cell is same as the assumed +ve direction, the emf of cell is taken as +ve other wise negative.

(II) For the path $\underline{AMLKA}$

$\cancel{I r + I_1 r_1 = E}$ $\quad -I_1 r_1 + E - I r = 0$

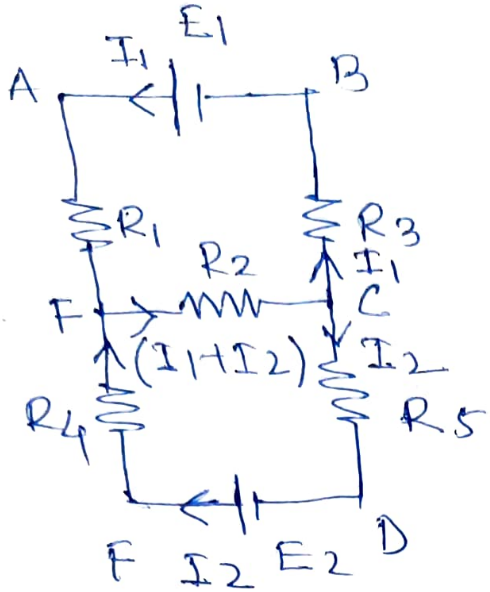
$E = I r + I_1 r_1$

(III) For the path $\underline{DEFGD}$

$\cancel{I_4 r_4 - I_3 r_3 = 0}$ $\quad -I_4 r_4 + I_3 r_3 = 0$

as there is no source of emf in the path.

- The direction of the current through r_4 is ~~positive~~ negative direction and the current r_3 which is ~~taken~~ in the opposite direction is taken as ~~negative~~ positive



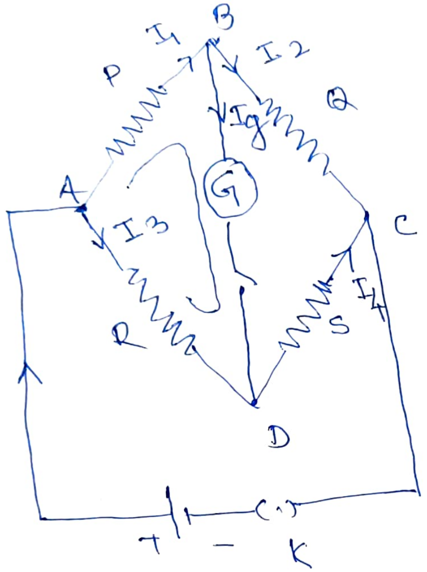
closed loop or path

$$-E_1 - I_1R_3 + I_2R_5 + E_2 + I_2R_4 - I_1R_1 = 0$$

$$\therefore -E_2 - E_1 = I_1R_1 + I_1R_3 - I_2R_4 - I_2R_5$$

$$= I_1(R_1+R_3) - I_2(R_4+R_5)$$

Application of Kirchhoff's law to Wheatstone's Bridge



~~$I_1P - I_gG + I_3R = 0$~~

Above figure represents the wheatstone's bridge ckt where P, Q, R and S are connected to form mesh. A cell is connected between points A and C and galvanometer is connected between point B and D. The current through the various branches are indicated in the figure. The current through the galvanometer is I_g and the resistance of the galvanometer is G.

Apply the first law
At the junction B

~~[illegible]~~

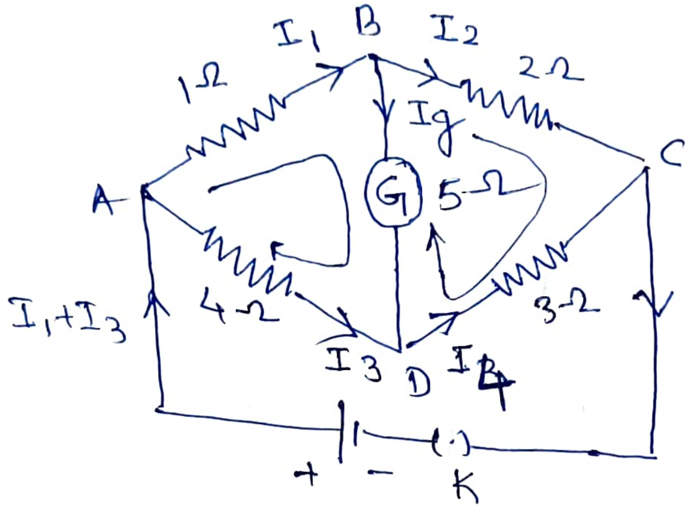
$$I_1 - I_2 - I_g = 0 \quad \text{——— (1)}$$

At the junction D

$$I_3 + I_g - I_4 = 0 \quad \text{—— (11)}$$

Apply second law to the mesh ABDA and ABCDA

Calculate the current passing through the galvano-meter in the following network



$E = 2V$

$r = 2\Omega$

Let the current in each branch of network be as shown in Fig.

Let us apply Kirchhoff's First law to junction B then we have

$I_1 - I_2 - I_g = 0$

or $I_1 = I_2 + I_g$

$\therefore I_2 = I_1 - I_g$ ——— (1)

Apply the same law to the junction D

$I_3 + I_g - I_4 = 0$

$I_4 = I_3 + I_g$ ——— (2)

Now apply the Kirchhoff's second law to loop ABDA, we get

$I_1 \cdot 1 + I_g \cdot 5 - I_3 \cdot 4 = 0$

or $I_1 + 5I_g - 4I_3 = 0$

$$I_1 = 4I_3 - 5I_g \qquad (3)$$

Again apply the kirchhoff's IInd law to loop BCDB, we have

$$2I_2 - 3I_4 - 5I_g = 0$$

or by using (1) and (2)

$$2(I_1 - I_g) - 3(I_3 + I_g) - 5I_g = 0$$

$$2I_1 - 2I_g - 3I_3 - 3I_g - 5I_g = 0$$

$$\cancel{2I_1 - 2I_g =}$$

$$2I_1 - 3I_3 - 10I_g = 0$$

using eqn (3)

$$2(4I_3 - 5I_g) - 3I_3 - 10I_g = 0$$

$$8I_3 - 10I_g - 3I_3 - 10I_g = 0$$

$$5I_3 - 20I_g = 0$$

devide by 5

$$I_3 - 4I_g = 0$$

$$I_3 = 4I_g \qquad (4)$$

Findly apply the same law to the loop ABCA

$$I_1 + 2I_2 + 2(I_1 + I_3) = 2$$

using eqn (1)

$$I_1 + 2(I_1 - I_g) + 2(I_1 + I_3) = 2$$

$$I_1 + 2I_1 - 2I_g + 2I_1 + 2I_3 = 2$$

$$5I_1 + 2I_3 - 2I_g = 2$$

using eqn (3)

$$5(4I_3 - 5I_g) + 2I_3 - 2I_g = 2$$

$$20I_3 - 25I_g + 2I_3 - 2I_g = 2$$

$$22I_3 - 27I_g = 2$$

by using eqn (4)

$$22 \times 4I_g - 27I_g = 2$$

$$88I_g - 27I_g = 2$$

$$61I_g = 2$$

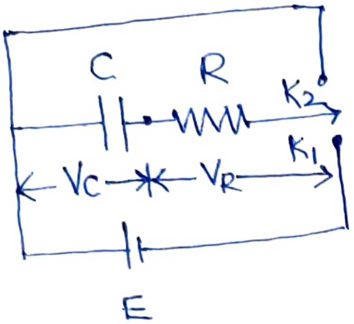
$$I_g = \frac{2}{61} \text{ amp.}$$

Charging and discharging of condenser through a resistance

Charging

~~$E \propto Q$~~ $Q \propto E$

~~$E = CQ$~~ $Q = EC$



Let a condenser of capacitance c be connected in series with resistance R and constant source of EMF E. The self inductance of the circuit is negligibly small. On closing key K1 the battery starts charging the condenser. A transient current passing through the ckt. so long as the condenser getting charged. Finally when the plates of the condenser acquire the charge $Q_0 = EC$, ~~the~~ the ~~current stop~~ current stop.

Let V_C and V_R be the potential difference across the condenser and resistance at any instant; Then $-V_C - V_R + E = 0$ clockwise direction.

$$E = V_C + V_R \quad \text{------------} \quad (1)$$

Let I be the current, and Q the charge on the condenser at any instant, then we have

$$I = \frac{dQ}{dt}$$

$$V_R = IR = R\frac{dQ}{dt}$$

and $\cancel{V_C} = \cancel{QC}$ $Q \propto V_C$, $Q = CV_C$

$$V_C = \frac{Q}{C}$$

then eqn. (1) becomes

$$R\frac{dQ}{dt} + \frac{Q}{C} = E$$

multiply by c through and rearranging

$$CR\frac{dQ}{dt} + \frac{Q}{\cancel{C}}\cdot\cancel{C} = CE$$

$$CR\frac{dQ}{dt} = CE - Q$$

$$CR\frac{dQ}{dt} = Q_0 - Q$$

$$-CR\frac{dQ}{dt} = -(Q_0 - Q)$$

$$-CR\frac{dQ}{dt} = -Q_0 + Q$$

Separating the variables

$$-CR\frac{dQ}{dt} = Q - Q_0$$

$$\frac{dQ}{Q - Q_0} = -\frac{dt}{CR}$$

on integration we get

$$\log_e(Q - Q_0) = -\frac{t}{CR} + c \quad \text{—— (2)}$$

c = constant of integration

to evaluate it we apply the initial condition that $t = 0,\ Q = 0$

hence

$$\log_e(-Q_0) = C$$

substitute this value in (2) we get

$$\log_e(Q-Q_0) = -\frac{t}{CR} + \log_e(-Q_0)$$

$$\log_e(Q-Q_0) - \log_e(-Q_0) = -\frac{t}{CR}$$

$$\log\left(\frac{Q-Q_0}{-Q_0}\right) = -\frac{t}{CR}$$

taking antilog

$$\left(\frac{Q-Q_0}{-Q_0}\right) = \exp^{-\frac{t}{CR}}$$

$$\left(\frac{Q}{-Q_0} - \frac{Q_0}{-Q_0}\right) = \exp^{-\frac{t}{CR}}$$

$$\left(-\frac{Q}{Q_0} + 1\right) = \exp^{-\frac{t}{CR}}$$

$$\left(1 - \frac{Q}{Q_0}\right) = \exp^{-\frac{t}{CR}}$$

multiply by Q_0

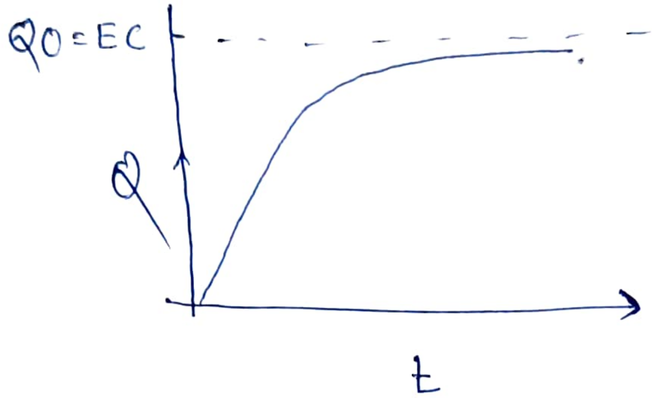
$$Q_0 - Q = Q_0 \exp^{-t/CR}$$

$$Q_0 - Q_0 e^{-\frac{t}{CR}} = Q$$

$$Q_0(1 - e^{-t/CR}) = Q$$

$$Q = Q_0 \left(1 - \exp^{-\frac{t}{CR}}\right)$$

the variation of Q and t is shown in fig



The rapidly with which, Q approaches the final value Q0 depend upon only CR which is time constant of CR circuit

when $t = CR$

$$Q = Q_0 \left(1 - \exp^{-1}\right)$$

$$= Q_0 \left(1 - \frac{1}{e}\right)$$

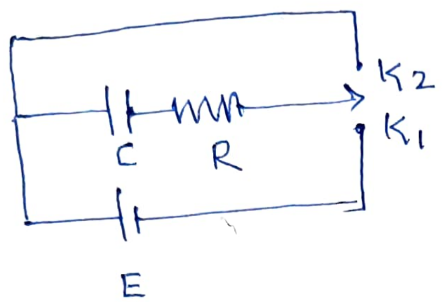
$$= Q_0 \left(1 - \frac{1}{2.72}\right)$$

$$= 0.63\, Q_0$$

Thus time constant is the time taken by condenser to acquires 0.63 of its maximum charge.

<u>Discharging</u>

ckt. diagram



On closing key K_1 condenser acquires the final steady charge $Q_0 = CE$ after some time. If now the key K_1 is open and K_2 is closed, the condenser discharges through the resistance R, the current strength being limited by R. (delaying the discharge.)

Let I be the current at any instant, so that $V_R = RI$. If the charge remaining on the plates of condenser is Q, the P.D across it is $V_C = Q/C$. Since there is no source of emf in this ckt, we have

$$V_R + V_C = E$$

$$E = 0$$

$$RI + \frac{q}{c} = 0$$

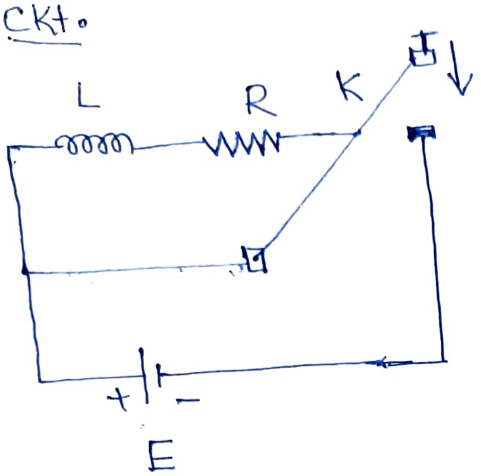
$$Q = Q_0 \times 0.368$$

$$Q = 0.368\, Q_0$$

The time required for the charge to reduce by 0.368 times the original charge on the plates of condenser is the time constant of the circuit.

Rise of Current in L·R Circuit.

Ckt.



Consider the circuit containing battery, key(K), inductance (L) and a resistance (R) joined in a series. The EMF of battery is E.

When key K is suddenly pressed, there is growth of current in the circuit and a back emf is induced. Suppose the current flowing through the circuit at any instant during the growth is I, Then

$$E = RI + L\frac{dI}{dt} \quad \text{------------} (1)$$

when the current reaches maximum value I_0 then the back emf, $L\frac{dI}{dt} = 0$ Then equation (1) will be

$$E = RI_0 \quad \text{------------} (2)$$

from equation (1) and (2)

$$RI_0 = RI + L\frac{dI}{dt}$$

$$RI_0 - RI = L\frac{dI}{dt}$$

$$R(I_0 - I) = L\frac{dI}{dt}$$

take $I_0 - I = x$ ------------ (3)

diff. w.r to time

$$-\frac{dI}{dt} = \frac{dx}{dt}$$

$\therefore$ $$Rx = -L\frac{dx}{dt}$$

$$\frac{dx}{x} = -\frac{R}{L}dt$$

integrate $\log x$ to base e

$$\log_e x = -\frac{R}{L}t + C$$

$$= -\frac{R}{L}t + C$$

where C is constant of integration

by using (3)

$$\log_e (I_0 - I) = -\frac{R}{L}t + C \quad \text{——— (4)}$$

when $t = 0$, $I = 0$ then

$$\log_e I_0 = C$$

substitute value of C in (4)

$$\log_e (I_0 - I) = -\frac{R}{L}t + \log_e I_0$$

$$\log_e (I_0 - I) - \log_e I_0 = -\frac{R}{L}t$$

$$\log_e \left(\frac{I_0 - I}{I_0}\right) = -\frac{R}{L}t$$

$$\left(\frac{I_0 - I}{I_0}\right) = e^{-\frac{R}{L}t}$$

$$\left(\frac{\cancel{I_0}}{\cancel{I_0}} - \frac{I}{I_0}\right) = e^{-\frac{R}{L}t}$$

$$\left(1 - \frac{I}{I_0}\right) = e^{-\frac{R}{L}t}$$

multiply I_0 through out

$$I_0\left(1 - \frac{I}{I_0}\right) = I_0 e^{-\frac{R}{L}t}$$

$$\left(I_0 - \frac{I}{\cancel{I_0}} \times \cancel{I_0}\right) = I_0 e^{-\frac{R}{L}t}$$

$$I_0 - I = I_0 e^{-\frac{R}{L}t}$$

$$I_0 - I_0 e^{-R/L\, t} = I$$

or $$I = I_0 - I_0 e^{-\frac{R}{L}t}$$

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right) \qquad (5)$$

the quantity L/R is called the time constant of the circuit

if $L/R = t$ then

$$I = I_0\left(1 - e^{-\frac{\cancel{R}}{\cancel{L}} \times \frac{\cancel{L}}{\cancel{R}}}\right)$$

$$= I_0\left(1 - e^{-1}\right)$$

$$= I_0\left(1 - \frac{1}{e}\right)$$

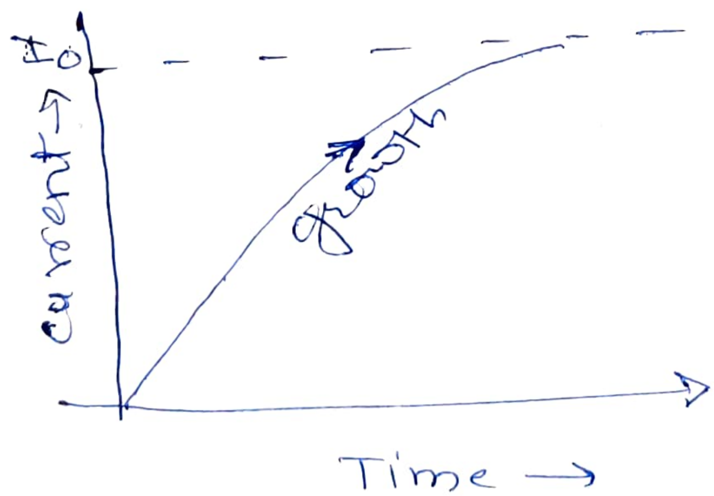
but $\frac{1}{e} = \frac{1}{2.718} = 0.368$

$\therefore I = I_0 (1 - 0.368)$

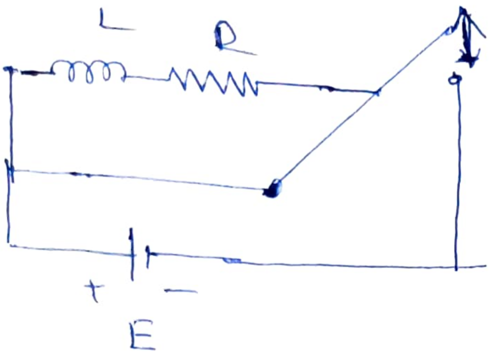
$= 0.632 I_0$

Thus the time constant L/R of a circuit is the time taken by ~~is~~ the current to grow from zero to 0.632 times the steady maximum value in the circuit.

The graph between current and the time at the time of growth of current is shown in the figure.



Decay of current in a circuit containing inductance and resistance



When the current in the circuit containing a resistance and inductance is suddenly switch off, an emf is again produced in this case $E=0$ and at any instant during decay

Apply Kirchhoff's second law

$$E - L\frac{dI}{dt} - IR = 0$$

$$E = IR + L\frac{dI}{dt}$$

$$0 = RI + L\frac{dI}{dt}$$

$$L\frac{dI}{dt} = -RI$$

$$\frac{dI}{I} = -\frac{R}{L}dt$$

integrating we get

$$\log_e I = -\frac{R}{L}t + C \quad \text{--- (1)}$$

where C is constant

when $t = 0$, $I = I_0$

$\therefore \log_e I_0 = C$

Substitute it in ①

$$\log_e I = -\frac{R}{L} t + \log_e I_0$$

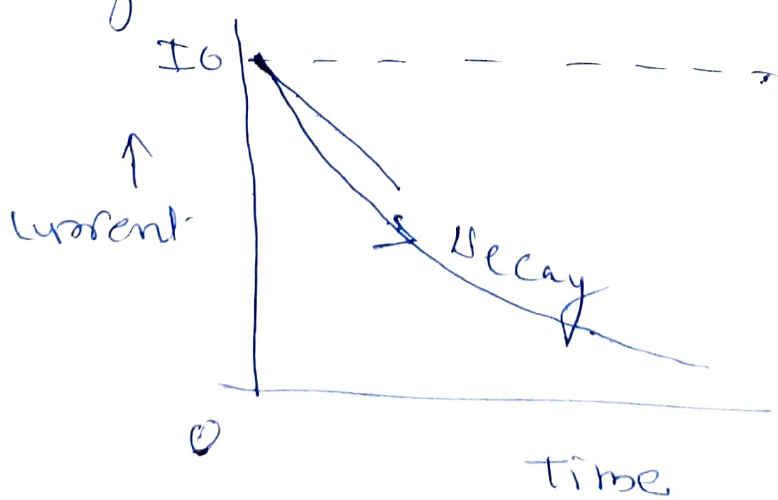
$$\log_e I - \log_e I_0 = -\frac{R}{L} t$$

$$\log_e \left(\frac{I}{I_0} \right) = -\frac{R}{L} t$$

$$\frac{I}{I_0} = e^{-\frac{R}{L} t}$$

$$I = I_0 e^{-\frac{R}{L} t} \quad \text{——— ②}$$

If the graph is plotted between current and time during decay, it will be an exponential curve the decay curve is just reflecting the growth curve



Here also L/R is the time constant

if $\frac{L}{R} = t$ then $I = I_0 e^{-\frac{R}{L} \frac{L}{R}} = I_0 e^{-1}$

$$I = I_0 \frac{1}{e}$$

$$\frac{1}{e} = 0.368$$

$$I = I_0 \times 0.368$$

$$= 0.368 I_0$$

Therefore time constant can be defined as the time taken by the current to fall from maximum value 0.638

Thus the time constant $\tau = \frac{L}{R}$ may also be considered as equal to the time for the current to decay by 0.63 of its initial maximum value.

Above ckt. a resistor R and Inductor L are connected in series. Such a device are used for measuring voltage difference across the resistor and inductor. A ~~switch~~ Key K can be connect battery of emf E into the circuit. Initially no current flow through the ckt when ~~switch~~ Key K is open. When the ~~switch~~ Key K is closed a current in the resistor start to ~~move~~ rise. If the inductor were not present the current is quickly rise to steady value. The inductor however gives induced emf which according to Lenz's law oppose the rise in current. That it is oppose the polarity of the battery emf. The current in the ckt. is due to two emf a constant emf is due to battery and variable emf of opposite sign due to inductance.